

# Charged elliptic flow at zero charge asymmetry

Misha Stephanov<sup>1,\*</sup> and Ho-Ung Yee<sup>1,2,†</sup>

<sup>1</sup>*Department of Physics, University of Illinois, Chicago, Illinois 60607, USA*

<sup>2</sup>*RIKEN-BNL Research Center, Brookhaven*

*National Laboratory, Upton, New York 11973-5000*

(Dated: April 25, 2013)

## Abstract

The difference between the flow ellipticities of oppositely charged pions,  $\Delta v_2 \equiv v_2[\pi^-] - v_2[\pi^+]$ , measured recently by STAR collaboration at RHIC shows a linear dependence on the event charge asymmetry  $A_{\pm} \equiv (N_+ - N_-)/(N_+ + N_-)$ :  $\Delta v_2(A_{\pm}) = \Delta v_2(0) + r A_{\pm}$  with a slope  $r > 0$  and a non-zero intercept  $\Delta v_2(0) > 0$  of order  $10^{-4}$ . We point out two novel mechanisms which could explain the non-zero value of the charged elliptic flow  $\Delta v_2$  at *zero* charge asymmetry  $A_{\pm} = 0$ , i.e., the non-zero positive intercept  $\Delta v_2(0)$ . Both effects are due to the electric fields created by the colliding ions. These fields have quadrupole asymmetry of the magnitude and the sign needed to account for the nonzero intercept  $\Delta v_2(0) > 0$  in the RHIC data. One of the mechanisms also involves chiral magnetic effect. This mechanism, although negligible at RHIC energies, may become important at LHC energies.

PACS numbers: 25.75.-q, 25.75.Ld, 25.75.Ag, 25.75.Gz

---

\*Electronic address: misha@uic.edu

†Electronic address: hyee@uic.edu

## I. INTRODUCTION

The recent analysis [1, 2] of STAR event-by-event measurements of the difference between the elliptic flows of positive and negative pions,  $\Delta v_2 \equiv v_2[\pi^-] - v_2[\pi^+]$ , finds an interesting behavior of this quantity as a function of the event charge asymmetry  $A_{\pm} \equiv (N_+ - N_-)/(N_+ + N_-)$ . As seen in FIG. 1,  $\Delta v_2$  depends linearly on  $A_{\pm}$ , i.e.,

$$\Delta v_2(A_{\pm}) = \Delta v_2(0) + r A_{\pm}, \quad (1)$$

with a slope  $r > 0$  and an intercept  $\Delta v_2(0) > 0$ . This observed positive slope agrees with the

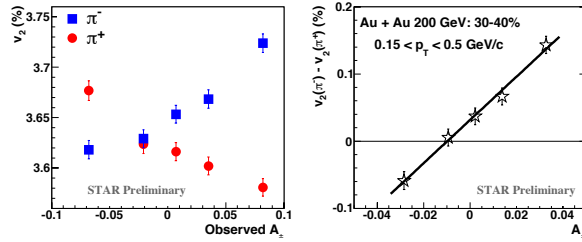


FIG. 1: The STAR data [1] on the elliptic flow difference between the positive and negative pions showing a linear dependence on the charge asymmetry  $A_{\pm} \equiv (N_+ - N_-)/(N_+ + N_-)$  with a finite positive intercept.

prediction made in Ref. [3, 4] based on the Chiral Magnetic Wave (CMW) [5, 6]. The CMW is a gapless sound-like mode of chiral charges (left-handed or right-handed) propagating along the direction of the magnetic field, with a dispersion relation

$$\omega = \pm v_{\chi} k - i D_L k^2 + \dots, \quad (2)$$

where the sign, and hence the direction of the propagation with respect to the magnetic field, depends on the chirality of the charge fluctuations. The velocity  $v_{\chi}$  is given by [5]

$$v_{\chi} = \frac{N_c e B}{4\pi^2} \alpha, \quad (3)$$

where  $\alpha$  is the inverse susceptibility of the chiral charge, and  $D_L$  – the longitudinal diffusion constant which depends on the microscopic transport properties of a given system. The mechanism described in Ref.[3, 4] is based on the observation (also made in Ref.[7]) that the initial *net* electric charge  $Q$  is a sum of the net charge carried by left-handed ( $Q_L$ ) and

right-handed ( $Q_R$ ) chiral carriers (quarks):

$$Q = Q_L + Q_R, \quad (4)$$

where on average  $Q_R = Q_L$ . In off-central heavy-ion collisions at RHIC energies the heavy ions can create a transient magnetic field of strength as large as  $B \sim 4m_\pi^2/e \sim 10^{19}\text{G}$  in the direction perpendicular to the reaction plane. The CMW moves chiral charges  $Q_L$  and  $Q_R$  along the axis of this magnetic field according to Eq. (2). Since  $Q_L$  moves in the direction opposite to that of  $Q_R$ , the net result will be an electric charge excess in the poles of the fireball and a depletion in the central region. See FIG. 2 for a schematic illustration of

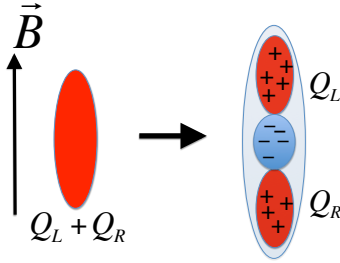


FIG. 2: The Chiral Magnetic Wave (CMW) evolution of the initial charge asymmetry (taken to be positive in this example) produces a net electric quadrupole moment responsible for a non-zero  $\Delta v_2 \equiv v_2[\pi^-] - v_2[\pi^+]$  (positive in this case). The collision plane is horizontal in this sketch.

the mechanism. Such a charge transport via CMW will result in an electric quadrupole moment whose magnitude is naturally proportional to the charge asymmetry. Combined with the subsequent radial flow this leads to the charged elliptic flow of pions,  $\Delta v_2$ , linear in  $A_\pm$  [3, 4]. A semi-realistic theoretical simulation performed in Ref.[3, 4] qualitatively and semiquantitatively agrees with the STAR data, including a non-trivial dependence on the impact parameter (centrality).

Although the CMW mechanism described above accounts for the nonzero positive slope in Fig. 1, the origin of non-zero positive intercept  $\Delta v_2(0)$  has not been so far understood to the same extent. An argument based on a simple quark coalescence model in Ref.[8] suggests that a positive contribution to the intercept may appear due to initial isospin asymmetry. In this paper we shall identify two different mechanisms which are more closely related to the CMW effect responsible for the nonzero slope. These mechanisms account for the sign as well as the magnitude of the intercept.

One way to look at the problem is to note that the nonzero intercept suggests the existence of an additional source of electric quadrupole moment, approximately independent of the charge asymmetry  $A_{\pm}$  and present even in the neutral plasma, i.e., at  $A_{\pm} = 0$ . We point out two sources of such a net positive (out of reaction plane) electric quadrupole moment. Both involve electric fields created by the heavy ions which are similar in strength to the magnetic fields responsible for the CMW. We describe the two mechanisms in the subsequent sections and make an order of magnitude estimate of their effect on the charged elliptic flow  $\Delta v_2(0)$ .

## II. SOURCE 1: ELECTRIC FIELD QUADRUPOLE

A highly relativistic heavy ion carries around itself a Lorentz contracted “pancake” of the electromagnetic field perpendicular to its velocity. The magnetic field lines are circular around the velocity direction and the heavy ions colliding at a finite impact parameter create a net magnetic field in the overlapping region along the direction perpendicular to the reaction plane. This field is responsible for the CMW.

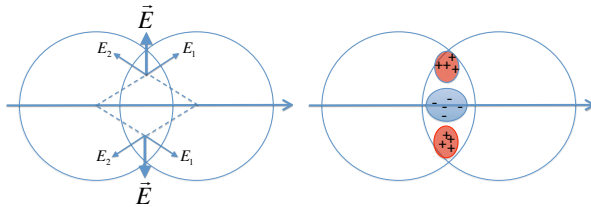


FIG. 3: The net electric field profile from the two heavy-ions (*left*), which leads to a non-zero electric quadrupole moment (*right*) whose magnitude is independent of the charge asymmetry  $A_{\pm}$  for small enough  $A_{\pm}$ . The reaction plane is horizontal.

On the other hand, the electric field from a single heavy ion is radially directed, perpendicular to the magnetic field at all points in space-time. The superposition of the electric fields from the two colliding ions in the overlapping region is described schematically in FIG. 3(left). The key feature of the effect we describe is that this electric field has a quadrupole component: the field pointing outward *perpendicular to* the reaction plane is stronger than the field pointing outward *in* the reaction plane due to partial cancellation of the in-plane components. Since the plasma (or pre-equilibrium state, such as glasma, at such early times) is conducting, there naturally arise net currents moving charges outward from

the reaction plane:  $\mathbf{J} \sim \mathbf{E}$ . One easily sees that this charge transport induces a net positive electric quadrupole moment along the out-of-plane axis (see FIG. 3) which, combined with subsequent radial collective flow, gives rise to  $\Delta v_2 > 0$  as in the CMW.

Since the conductivity is non-zero also in a *neutral* plasma, the effect is present even when  $A_{\pm} = 0$ . It is also clear from charge conjugation symmetry that the conductivity can only depend quadratically on the net charge density. Thus the dependence of the contribution of the quadrupole electric field effect to  $\Delta v_2$  in Eq. (1) on  $A_{\pm}$  is given by  $\Delta v_2(0) + \mathcal{O}(A_{\pm}^2)$ . The term quadratic in  $A_{\pm}$  is negligible for small enough  $A_{\pm}$  ( $A_{\pm}$  in FIG. 1 is  $\leq 0.04$ ).

We emphasize again that the magnitude of the electric field is similar to that of the magnetic field, which indicates that the order of magnitude of the induced quadrupole moment should be comparable to the one induced by the magnetic field via the CMW. This aspect and other similarities make this mechanism (and also the mechanism described in the following section) to be a natural possible explanation of the non-zero intercept in FIG. 1, if the nonzero slope is due to the CMW.

Let us make an order of magnitude estimate of the effect. The elliptic flow difference between positively and negatively charged particles is of order  $\Delta v_2 \sim \Delta Q / N_{\text{tot}}$  where  $\Delta Q$  is the total charge (in units of  $e$ ) transported by this mechanism and  $N_{\text{tot}}$  is the total multiplicity of charged particles. The value of  $\Delta Q$  can be estimated as  $\Delta Q \approx JA\tau_J$ , where  $J$  is the (units of  $e$ ) current density induced by the electric field,  $\tau_J$  is the lifetime of the current and  $A$  is the typical area of the fireball transverse to current during the lifetime of the electric field,  $\tau_E$ . The area transverse to the current at time  $\tau_E$  is given by the product of the transverse size, which for 30 – 40% central events we take to be of order of the radius of the nucleus  $R \sim 7$  fm, and of the longitudinal size, which is of order  $\tau_E$ :  $A = R\tau_E$ .

To estimate the current and its lifetime we need to consider two cases depending on whether the lifetime  $\tau_E$  of the electric field is shorter or longer than the typical (mean free) time between collisions  $\tau_{\text{free}}$ . Although the mechanisms of the charge transfer are different in the two cases, the relevant product,  $J \tau_J$ , turns out to be similar. As we shall see below, for relatively long-lived electric field,  $\tau_E \gg \tau_{\text{free}}$ , the magnitude of the current is determined by conductivity and  $J \sim \tau_{\text{free}}$ , while its lifetime is as long as that of the electric field  $\tau_J \approx \tau_E$ . On the other hand, for short-lived electric field,  $\tau_E \ll \tau_{\text{free}}$ , the magnitude of the current is determined by the lifetime of the field  $J \sim \tau_E$ , while its lifetime is limited by the mean free time  $\tau_J \approx \tau_{\text{free}}$ . In either case the relevant product  $J\tau_J \sim \tau_E\tau_{\text{free}}$ .

Let us now estimate the current  $J$ . First consider the case when the lifetime of the electric field  $\tau_E$  is much longer than the time between collisions  $\tau_{\text{free}}$ . This is the hydrodynamic regime and thus the current is given by Ohm's law  $J = \sigma E$ , where for the conductivity  $\sigma$  we could use a lattice QCD result from Ref. [9],

$$\sigma = 0.4eT \sum_{F=u,d,s} q_F^2 \approx 0.27eT. \quad (5)$$

When  $\tau_E \ll \tau_{\text{free}}$  we cannot use Ohm's law. But in this case we could estimate the current in the collisionless approximation. The contribution of a each quark or antiquark species  $f$  to the current is given by  $J_f = q_f n_f v_f$  (where  $f = u, d, s, \bar{u}, \bar{d}, \bar{s}$ ) where  $v_f$  is the drift velocity induced by the electric field by increasing momentum density of this component of the plasma by  $\pi_f = \tau_E e E q_f n_f$ . By Lorentz invariance  $v_f = \pi_f / w_f$ , where  $w_f$  is the momentum "susceptibility" (given by enthalpy  $\epsilon + p$  in equilibrium). Putting this together, and taking into account that, at  $A_{\pm} = 0$ , quark and antiquark of a given flavor contribute equally to the net current, i.e.,  $J_F = J_f + J_{\bar{f}} = 2J_f$ , we find:  $J = \sum_{F=u,d,s} J_F = 2eE\tau_E \sum_{F=u,d,s} q_F^2 n_F^2 / w_F$ , where the factor 2 in the last equation counts the equal contributions from quarks and antiquarks of a given flavor  $F$ . We can estimate  $n_F$  and  $w_F$  using equilibrium momentum distribution for massless quarks:  $n_F = 9\zeta(3)T^3/\pi^2$  and  $w_F = 7\pi^2 T^4/15$ . Thus we find

$$J \approx 0.26 \tau_E e E T^2 \sum_{F=u,d,s} q_F^2. \quad (6)$$

Comparing to Eq. (5) we can write  $J \approx 0.65(\tau_E T)\sigma E$ . Clearly, when  $\tau_E \approx \tau_{\text{free}}$ , the two equations should match, from which we could infer a rough estimate of the mean free time  $\tau_{\text{free}} \approx 1.5/T$  implicit in the lattice result Eq. (5). Using this as an estimate of the  $\tau_{\text{free}}$ , we can write the result in a form independent of which regime we consider:

$$J\tau_J \approx 0.17eET^2\tau_E\tau_{\text{free}}. \quad (7)$$

Thus

$$\Delta v_2 \approx \frac{\Delta Q}{N_{\text{tot}}} \approx \frac{J\tau_J A}{N_{\text{tot}}} \approx 0.17 \frac{eE T^2 \tau_E^2 \tau_{\text{free}} R}{N_{\text{tot}}}. \quad (8)$$

The electric field strength can be approximated as  $eE \approx \gamma Z e^2 / (4\pi R^2)$  where  $Z$  is the atomic number of the heavy nucleus,  $R$  is its radius, and  $\gamma = \sqrt{s}/(2 \text{ GeV})$  is the Lorentz contraction factor which we express in terms of the center of mass energy per colliding nucleon pair. The life-time of the electric field is roughly given by the longitudinally contracted

thickness of the colliding nucleus, which is  $\tau_E \approx 2R/\gamma$ , while  $\tau_{\text{free}} \approx 1.5/T$  as inferred from the lattice result (5). Putting this together and using  $Z \approx 80$  and  $R \approx 7 \text{ fm}$  we find

$$\Delta v_2 \approx 1. \times \frac{Z\alpha RT}{N_{\text{tot}}\gamma} \approx 10^{-4} \left( \frac{T}{400 \text{ MeV}} \right) \left( \frac{10^3}{N_{\text{tot}}} \right) \left( \frac{200 \text{ GeV}}{\sqrt{s}} \right), \quad (9)$$

which has the right order of magnitude to account for the intercept  $\Delta v_2(0) \approx 3.10^{-4}$  in Fig. 1.

### III. SOURCE 2: ELECTRIC QUADRUPOLE VIA CHIRAL MAGNETIC EFFECT

The other source of a net positive electric quadrupole moment arises through a combination of both  $\mathbf{E}$  and  $\mathbf{B}$ . Consider a P- and CP-odd quantity  $\mathbf{E} \cdot \mathbf{B}$ . From the electric field pattern in FIG. 3 as well as the magnetic field pointing upward perpendicular to the reaction plane, one easily recognizes that  $\mathbf{E} \cdot \mathbf{B}$  is positive in the upper half region, while it is negative in the lower half region as shown in FIG. 4(left). Via the triangle anomaly relation for the axial current  $J_A^\mu$ ,

$$\partial_\mu J_A^\mu = \frac{N_c e^2}{2\pi^2} \left( \sum_F q_F^2 \right) \mathbf{E} \cdot \mathbf{B}, \quad (10)$$

this implies a net positive axial charge created in the upper region, and net negative in the lower region. In the presence of the vertical magnetic field, the Chiral Magnetic Effect [10–12] acting on these axial charges induces an electric charge current (in units of  $e$ ),

$$\mathbf{J} = \frac{N_c e}{2\pi^2} \left( \sum_F q_F^2 \right) \mu_A \mathbf{B}, \quad (11)$$

from which it is easily seen that the net effect is a development of a positive electric quadrupole moment. See FIG. 4 (right) for a schematic explanation. To some extent, the mechanism is similar to that of the CMW in Ref.[3, 4], with a main difference being that the mechanism is independent of the initial charge asymmetry  $A_\pm$ .

To estimate the contribution of this mechanism to  $\Delta v_2$ , we start from the amount of axial charge density created by the non-zero  $\mathbf{E} \cdot \mathbf{B}$  during the lifetime  $\tau_E$  of the electromagnetic field,

$$J_A^0 \approx \frac{N_c e^2}{2\pi^2} \left( \sum_F q_F^2 \right) EB\tau_E \approx 0.1 (eE)(eB)\tau_E. \quad (12)$$

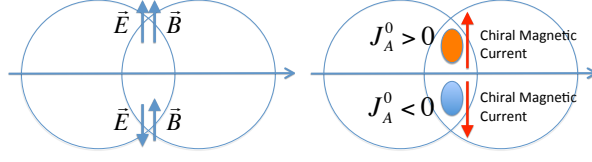


FIG. 4: The profile of  $\mathbf{E}$  and  $\mathbf{B}$  that leads to a creation of axial charges via triangle anomaly (*left*). The Chiral Magnetic Effect (CME) acting on these axial charges induces a net electric quadrupole moment (*right*).

This corresponds to an axial chemical potential of order  $\mu_A \approx j_A^0/\chi_A$  with the axial charge susceptibility  $\chi_A$  whose magnitude we can estimate assuming it is similar to the vector charge susceptibility  $\chi$  known from lattice QCD [13] to be about  $\chi \approx 1.0 T^2$ . The CME in Eq. (11) then produces a current

$$J = \frac{N_c}{2\pi^2} \left( \sum_F q_F^2 \right) \mu_A(eB) \approx 10^{-2} T^{-2} (eE)(eB)^2 \tau_E, \quad (13)$$

which would result in charge separation  $\Delta Q \approx JA\tau_J$  with  $A \approx R\tau_E$  as before. We could also expect that the duration of the chiral magnetic current  $\tau_J$  is approximately given by the duration of the magnetic field  $\tau_J \approx \tau_B$ . Putting this together and estimating the electric and magnetic fields again as  $eE \approx eB \approx \gamma Z\alpha/R^2$  one finds for  $\Delta v_2 \approx \Delta Q/N_{\text{tot}}$

$$\Delta v_2 \approx 10^{-2} \frac{(eE)(eB)^2 \tau_E^2 \tau_B R}{N_{\text{tot}} T^2} \approx 10^{-2} \frac{(Z\alpha)^3 \tau_E^2 \tau_B \gamma^3}{N_{\text{tot}} R^5 T^2} \approx 10^{-7} \left( \frac{400 \text{ MeV}}{T} \right)^2 \left( \frac{10^3}{N_{\text{tot}}} \right), \quad (14)$$

where we used  $\tau_E \approx \tau_B \approx 2R/\gamma$  and  $R \approx 7 \text{ fm}$  as before. We see that the magnetically induced electric quadrupole effect is negligible at RHIC energies compared to the direct electric quadrupole effect in Eq. (9). However, the direct effect in Eq. (9) decreases with energy  $\sqrt{s}$ , while the magnetically induced effect in Eq. (14) is energy independent.

In addition, the magnetically induced electric quadrupole is sensitive to the lifetime of the magnetic field  $\tau_B$ . For example, if this time turns out to be much greater than our estimate  $\tau_B = 2R/\gamma$  (due to the conductivity of the medium, as in Ref. [14]), that effect may be significantly larger than our estimate in Eq. (14) and could possibly compete with the direct electric dipole effect in Eq. (9) at sufficiently high  $\sqrt{s}$ :

$$\Delta v_2 \approx 10^{-5} \left( \frac{\tau_B}{2 \text{ fm}} \right) \left( \frac{eB}{1 \text{ fm}^{-2}} \right) \left( \frac{400 \text{ MeV}}{T} \right)^2 \left( \frac{10^3}{N_{\text{tot}}} \right). \quad (15)$$



Comparing this with the estimate of the direct electric quadrupole effect (9) we see that, while at top RHIC energy the direct electric quadrupole dominates, the two effects could possibly become comparable in magnitude ( $10^{-5}$ ) at LHC energies due to different  $\sqrt{s}$  dependence.

#### IV. DISCRETE SYMMETRIES

We end this note by discussing two discrete symmetries, the charge conjugation  $C$  and the  $180^\circ$  rotation (or reflection) in the transverse plane,  $R_\perp$ , useful for classifying possible mechanisms contributing to the observables: the slope  $r$  and the intercept  $\Delta v_2(0)$ . Under  $C$ ,

$$\Delta v_2(0) \rightarrow -\Delta v_2(0) \quad , \quad A_\pm \rightarrow -A_\pm \quad , \quad (16)$$

therefore  $r$  is  $C$ -even whereas  $\Delta v_2(0)$  is  $C$ -odd.

Since QCD is  $C$ -invariant and the source of  $C$ -violation is in the initial condition, one concludes that the physics of  $\Delta v_2(0)$  must have its origin in the initial charge asymmetry, such as the electromagnetic charge of the heavy-ions (or the isospin asymmetry as in Ref. [8]). On the other hand, the slope  $r$  could possibly receive contributions from other effects unrelated to the charge asymmetry of the initial conditions (as, e.g., in Ref. [15]).

Under the  $R_\perp$  rotation (reflection) in the transverse plane, both  $r$  and  $\Delta v_2(0)$  are even, since  $v_2$  is  $R_\perp$ -even. The pattern of the electric field is  $R_\perp$ -even, while the magnetic field pattern is  $R_\perp$ -odd. In conjunction with the  $C$ -parity discussion above, this tells us that the intercept  $\Delta v_2(0)$  can be linearly proportional to the electric field, but not the magnetic field. Indeed, our first source can be viewed as being linear in the electric field  $E$ , whereas the second source should be considered as an  $(EB)B \sim EB^2$  effect.

#### V. SUMMARY AND CONCLUSION

We identified two possible sources of the charged elliptic flow  $\Delta v_2(A_\pm)$  at zero charge asymmetry, i.e.,  $\Delta v_2(0)$ . The most straightforward source is the quadrupole pattern of the electric fields created by the relativistic heavy ions in the central overlap region. Although these fields last a short time, their magnitude is extremely large due to the well-known Lorentz contraction. Our order of magnitude estimates suggest that the contribution of this

effect at top RHIC energy is comparable with the observed value of  $\Delta v_2(0)$  and that it should be inversely proportional to the collision energy  $\sqrt{s}$ , dropping by an order of magnitude at LHC energies.

The simultaneous presence of electric and magnetic fields in the overlap region via the chiral magnetic effect provides another source of the quadrupole moment at zero charge asymmetry. The magnitude of this effect is less sensitive to the collision energy. It depends, however, on the lifetime of the magnetic fields, which could be longer than that of the electric fields. We estimate that this effect is negligible at RHIC energies but may become comparable with the direct electric quadrupole at LHC.

We stress that the two mechanisms we point out in this paper give rise to the same (positive out of plane) sign of the electric quadrupole moment, which makes the prediction for the sign of the effect robust. However, a reliable quantitative prediction of the magnitude would clearly require a realistic simulation including spatial variations and fluctuations of the electromagnetic fields [16–18].

## Acknowledgments

We thank Adam Bzdak, Olga Evdokimov, Jinfeng Liao, Todd Springer, and Yi Yin for helpful discussions. The work of M.S. is supported by the DOE Grant No. DE-FG02-01ER41195.

- 
- [1] G. Wang [STAR Collaboration], “Search for Chiral Magnetic Effects in High-Energy Nuclear Collisions,” [arXiv:1210.5498 [nucl-ex]].
  - [2] H. Ke [STAR Collaboration], “Charge asymmetry dependency of  $\pi^+/\pi^-$  elliptic flow in Au + Au collisions at  $\sqrt{s_{NN}} = 200$  GeV,” J. Phys. Conf. Ser. **389**, 012035 (2012).
  - [3] Y. Burnier, D. E. Kharzeev, J. Liao and H. -U. Yee, “Chiral magnetic wave at finite baryon density and the electric quadrupole moment of quark-gluon plasma in heavy ion collisions,” Phys. Rev. Lett. **107**, 052303 (2011).
  - [4] Y. Burnier, D. E. Kharzeev, J. Liao and H. -U. Yee, “From the chiral magnetic wave to the charge dependence of elliptic flow,” [arXiv:1208.2537 [hep-ph]].

- [5] D. E. Kharzeev and H. -U. Yee, “Chiral Magnetic Wave,” *Phys. Rev. D* **83**, 085007 (2011).
- [6] G. M. Newman, “Anomalous hydrodynamics,” *JHEP* **0601**, 158 (2006).
- [7] E. V. Gorbar, V. A. Miransky and I. A. Shovkovy, “Normal ground state of dense relativistic matter in a magnetic field,” *Phys. Rev. D* **83**, 085003 (2011).
- [8] J. C. Dunlop, M. A. Lisa and P. Sorensen, “Constituent quark scaling violation due to baryon number transport,” *Phys. Rev. C* **84**, 044914 (2011).
- [9] H. -T. Ding, A. Francis, O. Kaczmarek, F. Karsch, E. Laermann and W. Soeldner, “Thermal dilepton rate and electrical conductivity: An analysis of vector current correlation functions in quenched lattice QCD,” *Phys. Rev. D* **83**, 034504 (2011).
- [10] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, “The Effects of topological charge change in heavy ion collisions: ‘Event by event P and CP violation’,” *Nucl. Phys. A* **803**, 227 (2008).
- [11] K. Fukushima, D. E. Kharzeev and H. J. Warringa, “The Chiral Magnetic Effect,” *Phys. Rev. D* **78**, 074033 (2008).
- [12] D. T. Son and A. R. Zhitnitsky, “Quantum anomalies in dense matter,” *Phys. Rev. D* **70**, 074018 (2004).
- [13] P. Hegde, F. Karsch and C. Schmidt, “Calculating Quark Number Susceptibilities with Domain-Wall Fermions,” *PoS LATTICE* **2008**, 187 (2008).
- [14] K. Tuchin, arXiv:1301.0099 [hep-ph].
- [15] A. Bzdak and P. Bozek, “Contributions to the event-by-event charge asymmetry dependence for the elliptic flow of  $\pi^+$  and  $\pi^-$  in heavy-ion collisions,” arXiv:1303.1138 [nucl-th].
- [16] A. Bzdak and V. Skokov, “Event-by-event fluctuations of magnetic and electric fields in heavy ion collisions,” *Phys. Lett. B* **710**, 171 (2012) [arXiv:1111.1949 [hep-ph]].
- [17] W. -T. Deng and X. -G. Huang, “Event-by-event generation of electromagnetic fields in heavy-ion collisions,” *Phys. Rev. C* **85**, 044907 (2012) [arXiv:1201.5108 [nucl-th]].
- [18] J. Błochynski, X. -G. Huang, X. Zhang and J. Liao, “Azimuthally fluctuating magnetic field and its impacts on observables in heavy-ion collisions,” arXiv:1209.6594 [nucl-th].